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FOREST TAXATION AND ROTATION AGE UNDER PRIVATE AMENITY VALUATION: NEW RESULTS***

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ABSTRACT: This paper studies the impact of harvesting, property and profit taxes on private rotation age in an ongoing rotations model, where the private landowner is interested not only in the present value of harvest revenue, but also in the private amenity services provided by the forest stand. The main finding of the paper is that conventional wisdom about the rotation effects of forest taxes, distilled from the Faustmann model, predominantly ceases to hold. This is because forest taxes distort the relative profitability of timber and amenity production in a way that is sensitive to the precise nature of amenity valuation. Therefore, the design of forest tax policy necessitates good knowledge of the landowner's objective function concerning the type of amenity services.

Key words: Private rotation, amenity services, forest taxation.

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TIIVISTELMÄ: Paperissa tutkitaan metsäverotuksen vaikutusta metsän optimaaliseen kiertoaikaan, kun yksityismetsänomistaja on kiinnostunut hakkuutulojen lisäksi myös metsien tuottamista virkistyspalveluista. Tarkastelun kohteena on useita vaihtoehtoisia veromuotoja: myyntitulovero, yksikkövero, voittovero sekä erilaisia omaisuusveroja. Tutkimuksen päätulos on, että perinteinen, Faustmannin kiertoaikamallista johdettu viisaus metsäverojen kiertoaikavaikutuksista, lakkaa pääsääntöisesti pitämästä paikkaansa. Syynä tähän on, että nyt metsäverot vääristävät puuntuotannon ja virkistyskäytön suhteellista edullisuutta riippuen siitä, kuinka virkistyskäytön arvostus kehittyy puuston ikääntymisen myötä. Tämän vuoksi metsäveropolitiikan suunnittelu edellyttää hyvää tietoa metsänomistajien tavoitefunktiosta virkistyspalvelujen luonteen suhteen.

Avainsanat: yksityinen kiertoaika, virkistyspalvelut, metsäverotus

CONTENTS

1 INTRODUCTION

The impacts of several kinds of forest taxes on privately optimal rotation age has been extensively studied in the Faustmann model, where the forest owner maximizes the net present value of harvest revenue [4,9]. It may be the case, however, that private landowners are interested not only in the present value of net harvesting revenues, but also in the amenity services provided by forest stands. How does the amenity valuation affect the privately optimal rotation? This analysis was started by Hartman [8], and later Bowes and Krutilla [2], as well as Johansson and Löfgren [10], showed that if amenity services increase with the age of the forest stand, allowing for them will lengthen the rotation age at the interior solution. Are the effects of forest taxes on private rotation age sensitive to the question of introducing amenity services as a part of the objective function of the private landowners? We do not know since these effects have not to our knowledge been studied. The purpose of this paper is to investigate the impact of standard forest taxes on private rotation age in the Hartman model, and to compare the results with those obtained from the basic Faustmann model.

We consider three broad classes of forest taxes. One class consists of (i) *property taxes* levied on land value. Within this class two typically analyzed taxes on property are of the lump-sum type. *The site productivity tax* is paid annually and is based on the yield potentiality of a given site irrespective of the actual harvests or standing timber [11]. Another lump-sum type property tax is a proportional tax on the land value, called *a site value tax* [4,5]. *A* property tax may be also levied on the value of trees, and is often called *timber tax* [4]. Another forest tax class consists of (ii) *harvest taxes*. The most common version of harvest taxes is *the yield tax*, which is levied on the harvest revenue. Alternatively, *a unit tax* levied on the timber volume harvested can be used. Finally, we have (iii) *a profit tax* levied on the net timber revenue the landowner gets from the forests.

In section 2, we briefly revisit the standard results concerning the impacts of taxes on private rotation age in the basic Faustmann model to yield a yardstick for further analysis. In section 3, forest taxation is analyzed in the Hartman framework, in which the private landowner produces timber and amenity services as joint production. In section 4, there are some concluding remarks.

2 FOREST TAXATION IN THE FAUSTMANN MODEL – A RECAPITULATION

We consider a representative private landowner who begins with bare land, plants trees and clear cuts under known and constant timber price and interest rate so as to maximize the net present value of harvest revenue over infinite cycles of rotations. This is given by

$$V \quad = \quad (1-e^{-rT})^{-1}V^{J}, \qquad \textbf{(1)}$$

where $V^{J} = pf(T)e^{-rT} - c$, p is stumpage price, $f(T)$ is the growth of the stand as a function of its age T and which has the conventional convex-concave properties ($f'(T)>0$, and $f''(T)>0$ for $t<\bar{t}$ and $f''(T)<0$, $t>\bar{t}$, where $\bar{t}$ is the inflexion point of the growth function) and c denotes the regeneration cost.

- **Harvest taxes**

If the government levies the yield (τ) or the unit tax (t) on harvesting, then the net present value of revenue from a single rotation can be written as

$$\hat{V}^{J}(\tau,t) = \hat{p}f(T)e^{-rT} - c, \qquad \textbf{(2)}$$

where $\hat{p} \equiv p(1-\tau)-t$ denotes the after-tax stumpage price with the yield and unit taxes. In what follows we denote the after-tax present value in various cases by $\hat{V}$. Substituting the RHS of equation (2) for V^{J} in equation (1) and differentiating it with respect to the rotation age T, gives the following first-order condition for **a** privately optimal rotation period

$$\hat{V}_{T}(\tau,t) = \hat{p}(f'(T) - rf(T)) - rV = 0.^{1} \qquad \textbf{(3)}$$

The effects of harvest taxes can be obtained from $T_{\tau}^{F} = (-\hat{V}_{TT})^{-1}\hat{V}_{T\tau}$ and $T_{t}^{F} = (-\hat{V}_{TT})^{-1}\hat{V}_{Tt}$, where $\hat{V}_{TT}<0$ and $\hat{V}_{T\tau} = -p(f'(T) - rf(T) - re^{-rT}(1-e^{-rT})^{-1}) > 0$, and $\hat{V}_{Tt} = \dfrac{\hat{V}_{T\tau}}{p} > 0$.[2] Hence we have

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[1] We denote the partial derivatives by primes for functions with one argument and by subscripts for functions with many arguments. Hence, e.g. $f'(T) = \frac{\partial f(T)}{\partial T}$ for $f(T)$, while $A_{x}(x,y) = \frac{\partial A(x,y)}{\partial x}$ for $A(x,y)$, etc.

[2] The notation T^{F} refers to the rotation age in the Faustmann model (and in the next section

$T_\tau^F, T_t^F > 0$. Both harvest taxes will lengthen the private rotation age, since they affect like a decrease in the net stumpage price.

- **Property taxes**

We start by first exploring two kinds of property taxes, levied either directly on the value of forestland, or indirectly on administratively set site quality classification values of forestland.[3] In the former case of the site value tax, we denote the annual tax payment by *b* and define its present value as

$$\int_0^\infty be^{-rs}ds = \frac{b}{r}. \quad \textbf{(4)}$$

If the proportion of the value of the forestland delivered in taxes is β, then we have from equation (4) $b = r\beta V$, so that the after-tax value of the forestland is now $\hat{V}(\beta) = (1-\beta)V$. Maximization of the pre- and after-tax site value with respect to rotation age T reveals that $\hat{V}_T(\beta) = V_T = 0$, so that $\hat{V}_{T\beta} = 0$ and $T_\beta^F = 0$. The site value tax β, will have no effect on rotation age.

The property tax can also be based on the site productivity value of the land, denoted by a(i), where i refers to site index i of the land. The after-tax land value is given by

$$\hat{V}(a(i)) = V - \frac{a(i)}{r}. \quad \textbf{(5)}$$

where a(i)/r is the present value of the site productivity tax. Since $\hat{V}_T(a(i)) = V_T = 0$ we can conclude the rotation age is not affected by the site productivity tax.

In the case of a property tax on timber at a rate α, levied annually on the stumpage value of growing timber volume, the objective function of the landowner can be written as

$$\hat{V}(\alpha) = (1-e^{-rT})^{-1}\left[\; V^J - \alpha\int_0^T pf(s)e^{-rs}ds \;\right]. \quad \textbf{(6)}$$

The first-order condition with respect to rotation age *T* is now given by

$$\hat{V}_T(\alpha) = pf'(T) - rf(T) - rV - \alpha(pf(T) - rU) = 0, \quad \textbf{(7)}$$

T^H to that in the Hartman model).

[3] See e.g. [9].

where $U = (1-e^{-rT})^{-1}\int_0^T pf(s)e^{-rs}ds$ denotes the present value of annual timber earnings, and rV refers to the value of the forestland in the absence of timber tax.

The effect of timber tax α on the private rotation age turns out to depend on the sign of the expression $(pf(T)-rU)$. Under the assumption $f'(T)>0$ over the relevant economic region we have the following

Lemma 1. $pf(T)-rU>0$, when $f'(T)>0$.

Proof. See appendix 1.

Lemma 1 says that when $f'(T)>0$, the value of timber stock at the harvest time $pf(T)$, is greater than the opportunity cost of harvest rU, given by the interest on the present value of annual timber earnings. The first-order condition (7) together with Lemma 1 yields $\hat{V}_{T\alpha}(\alpha) = -(pf(T)-rU)<0$, so that $T_\alpha^F<0$. The property tax on timber shortens the private rotation age.

- **Profit tax**

If the proportional profit tax θ is used, the net harvest revenue in the absence of other taxes can be written as $\hat{V}^J(\theta) = (1-\theta)V^J$ for a single rotation, and as $\hat{V}(\theta) = (1-\theta)V$ for ongoing rotations. The profit tax will have no effect on the private rotation age.

To summarize, we have[4]

Proposition 1. *The effects of forest taxes on the private rotation age in the basic Faustmann model are*

- *As harvest taxes, the yield tax and the unit tax taxes will lengthen the private rotation age, i.e.,* T_τ^F, $T_t^F>0$.
- *As annual lump-sum taxes, the site value tax and site productivity tax, as well as the profit tax, will have no effect on the private rotation age, i.e.,* $T_\beta^F = T_{a(i)}^F = T_\theta^F = 0$.
- *Timber tax, levied annually on stumpage value of growing timber, will shorten the private rotation age, i.e.,* $T_\alpha^F<0$.

[4] See [4] for timber tax, and [10] for other forest taxes.

Interpretation of Proposition 1 is straightforward. Harvest taxes reduce the net timber price. This decreases both the marginal return of delaying harvest and the marginal opportunity cost of delaying harvest. Since the latter effect dominates, the private rotation age is lengthened. Lump-sum property and profit taxes decrease the land value, but leave the relationship between the marginal return and the marginal opportunity cost unchanged. Hence, the rotation period remains unchanged as well. Finally, the timber tax decreases both the value of timber stock at harvest time and the opportunity cost of harvesting, with the former effect dominating. The rotation age will become shorter.

3 FOREST TAXATION IN THE HARTMAN MODEL

From now on we assume that the private landowner values both the net harvest revenue and the amenity services from forest stands.[5] Based on [8] we postulate the following quasi-linear objective function for one rotation in the absence of taxes

$$W^J = V^J + \int_0^T F(s)e^{-rs}ds, \quad \textbf{(8)}$$

where V^J describes the net present value of harvest for a single rotation and the integral term of equation [8] describes the present value of amenity services from the forest stand under a single harvest cycle of length T, and $F(s)$ is the flow of amenities for the stand of age s. The value of the quasi-linear utility function (8) over an infinite cycle of rotations can be written as

$$W = V + E, \quad \textbf{(9)}$$

where $V = (1-e^{-rT})^{-1}V^J$ and $E = (1-e^{-rT})^{-1}\int_0^T F(s)e^{-rs}ds$ describe the net present value of harvest revenue and the present value of amenity services over all rotations, respectively.

The first-order condition for the maximization of (9) can be expressed as

$$W_T = V_T + F(T) - rE = 0, \quad \textbf{(10)}$$

where $V_T = pf'(T) - rpf(T) - rV$. Hence, equation (10) can be written as

$$W_T = pf'(T) - rpf(T) - rV + F(T) - rE = 0. \quad \textbf{(10')}$$

According to (10'), the private landowner equates the marginal benefit of delaying the harvest to age T, defined by $pf'(T) + F(T)$, to the marginal opportunity cost of delaying the harvest, defined by $rpf(T) + r(V + E)$.

The second-order condition is

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5 Papers including [1,12,13] provide in various ways some indirect empirical evidence in favor of the hypothesis that private forest owners are actually interested not only in harvest revenue but also in amenity values of their forests. We must add, however, that precise hypotheses associated with the amenity services have not been studied empirically. This is an interesting, though challenging, area for further research.

$$W_{TT} = pf''(T) - rpf'(T) + F'(T) < 0\,, \qquad \textbf{(11)}$$

which we assume to hold in what follows.[6] The first-order condition (10) implicitly defines the Hartman rotation age T^H, which maximizes the sum of net present value of harvests and amenity valuation in terms of exogenous parameters.

To see how the privately optimal rotation age changes relative to the basic Faustmann model due to amenity valuation, it is useful to characterize first how the relative size between the amenity benefits at the harvest time and its opportunity cost depend on the precise nature of the private valuation function. This is shown in the following Lemma 2.[7]

Lemma 2. $F(T) - rE \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \;\; as \;\; F'(T) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$

Proof. See Appendix 2.

According to Lemma 2, if the amenity valuation does not change with the age of the forest stand ($F'(T) = 0$), then $F(T) = rE$ and the Faustmann and Hartman rotations coincide. However, if the valuation increases (decreases) with the age of the forest stand, then the timber production part, V_T, of equation (10) will be negative (positive), so that the Hartman rotation period is longer (shorter) than the Faustmann rotation period. This result turns out to be important when we evaluate the impact of harvest, property and profit taxes on the rotation age in the Hartman framework. These impacts have not been studied in the literature thus far. Englin and Klan have analyzed forest taxation in the framework, which in principle allows for amenity services [7]. They assume, however, that the private landowner does not account for the amenity valuation in his behavior so that actually they study the effects of forest taxes on the private rotation age in the Faustmann model.

- **Harvest taxes**

If the government levies the yield or the unit tax on harvesting, the net revenue from a single rotation can be expressed as in the previous section as $\hat{V}^J(\tau,t) = \hat{p}f(T)e^{-rT} - c$, while the amenity part E remains unchanged. Differentiating equation (9) in the presence of harvest taxes τ and t with respect to T yields

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6 This may not always hold when the amenity valuation increases with the age of a forest stand, i.e. when $F'(T) > 0$. Further discussion and detailed numerical analyses of various alternative cases can be found in [3,8,14,15].

7 The content of this Lemma 2 can be found also in [2, p.539, and 10].

$$\hat{W}_T(\tau,t) = \hat{p}(f'(T) - rf(T)) - rV + F(T) - rE = 0\,. \qquad \textbf{(12)}$$

The impact of harvest taxes on the private rotation age is given by $T_x^H = (-\hat{W}_{TT})^{-1}\hat{W}_{Tx}$, for $x = \tau, t$, so that it depends on the sign of $\hat{W}_{Tx}$. We get

Proposition 2. *The effects of harvest taxes on the private rotation age in the Hartman model depend both on the regeneration costs and on the amenity valuation function as follows:*

$$T_\tau^H, T_t^H \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad as \;\; rc(1-e^{-rT})^{-1} + F(T) - rE \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0\,.$$

Proof. See Appendix 3.

Proposition 2 offers new insight into the rotation effects of harvest taxes by pointing out the crucial role of both the nature of the amenity valuation and regeneration costs. According to Lemma 2, $F'(T) > 0$ implies $\hat{V}_T(\tau,t) < 0$. In this case harvest taxes have an effect similar to that of a reduction in the net timber price with no effect on the amenity part of equation (12), making the amenity production relatively more profitable. Therefore, the landowner lengthens the rotation age, as in the basic Faustmann model. If the amenity valuation does not change with T, then at the optimum $\hat{V}_T(\tau,t) = 0$, implying that the harvest taxes have not only the same qualitative, but also the same quantitative impact on the rotation period as in the Faustmann model, because amenity production does not matter at the margin. Finally, in the case of $F'(T) < 0$, the rotation age is shorter than that of the Faustmann model. Now timber production has become less profitable and provided that the regeneration costs are “small enough”, the landowner shifts towards amenity production by shortening the rotation age.[8]

- **Property taxes**

Recalling equations (4) and (5) from the previous section we can express the landowner’s objective functions for the site value tax and the site productivity tax, respectively, as

[8] Bowes and Krutilla have analyzed the comparative statics of timber price in the Hartman model in the special case of zero regeneration costs showing that timber price shortens the rotation age if the Hartman rotation is longer than the Faustmann rotation, i.e. if $F'(T) > 0$ [2, p. 540-541].

$$\hat{W}(\beta) = (1-\beta)V + E\,, \tag{13a}$$

$$\hat{W}(a(i)) = V - \frac{a(i)}{r} + E\,. \tag{13b}$$

For the site value tax we have $\hat{V}(\beta) = (1-\beta)V$, and the first-order condition for the maximization of (13a) is given by

$$\hat{W}_T(\beta) = (1-\beta)(pf'(T) - rpf(T) - rV) + F(T) - rE = 0\,. \tag{14}$$

Differentiating equation (14) with respect to β and utilizing Lemma 2 yields

$$\hat{W}_{T\beta}(\beta) = -(pf'(T) - rpf(T) - rV) \geq (<) 0 \;\; as \;\; F'(T) \geq (<) \; 0\,. \tag{15}$$

Using equation (15) and the fact that sign (T_β^H) =sign ($W_{T\beta}$), we get

$$T_\beta^H \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \;\; as \;\; F'(T) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0\,. \tag{16}$$

According to equation (16) the site value tax is neutral only when the amenity valuation is site-specific, i.e., when $F'(T) = 0$. If $F'(T) > (<) \; 0$, then a rise in the site value tax makes amenity production relatively more (less) profitable. Consequently, the landowner lengthens (shortens) the rotation age.

For the site productivity tax, the first-order condition is $\hat{W}_T(a(i)) = V_T + F(T) - rE = 0$. The site productivity tax is neutral with respect to the landowner's amenity valuation, because it tax does not distort the relative profitability of timber and amenity production.

In the case of the property tax on the stumpage value α, the objective function of the landowner is given by

$$\hat{W}(\alpha) = V - \alpha(1-e^{-rT})^{-1}\int_0^T pf(s)e^{-rs}ds + E\,. \tag{17}$$

The first-order condition for the privately optimal rotation age is

$$\hat{W}_T(\alpha) = pf'(T) - rpf(T) - rV - \alpha(pf(T) - rU) + F(T) - rE = 0\,. \tag{18}$$

Differentiating equation (18) with respect to α yields $\hat{W}_{T\alpha}(\alpha) = -(pf(T) - rU) < 0$ due to Lemma 1. Hence, the timber tax shortens rotation age irrespective of the sign of $F(T) - rE$, because the sign of the timber production part in (18) does not determine the sign of $\hat{W}_{T\alpha}(\alpha)$, which holds for any timber growth function with $f'(T) > 0$. The interpretation is the same as in the Faustmann model.

- **Profit tax**

Finally, let us analyze how the profit tax, neutral in the basic Faustmann model, affects private rotation under in situ preferences. The private landowner now maximizes $\hat{W}_T(\theta) = (1-\theta)V + E$. Choosing T optimally gives

$$\hat{W}_T(\theta) = (1-\theta)V_T + F(T) - rE = 0. \qquad \textbf{(19)}$$

Differentiating this with respect to θ and utilizing Lemma 2 yields

$$W(\theta)_{T\theta} = -(pf'(T) - rpf(T) - rV) \geq (<)\ 0 \ \ as \ \ F'(T) \geq (<)\ 0. \qquad \textbf{(20)}$$

Summarizing, we have obtained the following results for property and profit taxes:

Proposition 3. *The effects of property taxes on private rotation age in the Hartman model are*

- *The site value tax levied on the value of forestland and the profit tax affect the private rotation age a priori ambiguously, and depend on the precise nature of the amenity valuation function as follows*:

$$T_\beta^H \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \ \ and \ \ T_\theta^H \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \ \ as \ \ F'(T) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$$

- *The site productivity tax will have no effect on private rotation age, i.e.,* $T_{a(i)}^H = 0$.
- *The timber tax levied on the stumpage value of growing timber will shorten the private rotation age, i.e.,* $T_\alpha^H < 0$.

While timber and site productivity taxes affect private rotation age in the Hartman model just as in the Faustmann model, the site value tax remains neutral only in the case of site-specific amenities. In other cases it may either lengthen or shorten rotation depending on whether the amenity valuation increases or decreases with the age of the forest stand respectively. An annual lump-sum tax decreases that part of land value, which comes from the timber production, but does not affect the amenity valuation part, so that amenity production becomes more profitable relative to timber production. Therefore, a shift towards amenity production implies a longer (shorter) rotation age if amenity valuation increases (decreases) with the age of the forest stand. The effect of profit tax has the same interpretation as that of the site value tax.

To recapitulate, the effects of forest taxes in the Hartman model depend both on how they change the relative profitability of timber vis-a'-vis amenity production and on how the amenity valuation is related to the age of the forests. If amenity valuation does not change with the age of the forest stand, the effects of harvest taxes will be similar to those in the Faustmann model. Likewise, the effects of the profit and lump sum tax on the value of forestland will depend on the precise nature of amenity valuation function. These remain neutral taxes only when amenity valuation does not depend on the age of the stand, i.e. when amenity valuation is site-specific.

4 CONCLUDING REMARKS

In this paper we have taken the basic Faustmann model as the benchmark and studied the impact of harvesting, property and profit taxes on private rotation age in the Hartman model, where the private landowner is interested not only in the net present value of harvest revenue but also in the amenity services provided by the forest stand. The main finding of the paper is that if amenity services matter in the landowner's decision making, the conventional wisdom on the rotation impacts of various harvest, property and profit taxes on private rotation age, distilled from the basic Faustmann model, predominantly ceases to hold. This is because forest taxes distort the relative profitability of timber and amenity production in a way that is sensitive to the precise nature of the amenity valuation. Therefore the optimal design of forest tax policy necessitates good knowledge of the landowner's amenity valuation function.

When the impacts of various forest taxes on private rotation age in a partial equilibrium framework are now known also in the Hartman model, the next question to ask is: What would be the optimal structure of forest taxation from the point of view of society?

Gamponia and Mendelsohn have studied the relative efficiency of the yield and property taxes from the point of view of minimizing the excess burden due to their distortionary effect on private rotation age in the Faustmann model [6]. From the viewpoint of collecting tax revenue, a neutral forest tax has been usually regarded as the best, because it does not distort the private landowner's decision making on when to cut the trees. In addition to this fiscal motive, forest taxes can also be justified from another point of view. If society values the amenity services provided by the private forest stands more than the landowner, then private rotation age is too short from the point of view of the society. Under these circumstances a forest tax that lengthens the private rotation period, can be a useful tool to achieve a socially optimal level of joint production of timber and amenities. A general welfare analysis of forest taxation in an ongoing rotations framework with amenity services is an interesting topic for further research.

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LIST OF SYMBOLS:

$f(T)$ growth function of a stand

T rotation age

p real stumpage price

c real regeneration cost

r real interest rate

τ yield tax (levied on the stumpage value of timber harvested)

t unit tax (levied on the volume of timber harvested)

b annual lump-sum tax payment (levied on the landowner)

β site value tax (annual lump-sum tax b related to the value of the land)

a(i) site productivity tax for site i (a lump-sum tax levied on the productivity of site i)

α timber tax (levied on the stumpage value)

θ profit tax (levied on the net harvest revenue)

$F(T)$ amenity valuation function

T^F Faustmann rotation age

T^H Hartman rotation age

V^J the net present value of harvest revenue for a single rotation

V the net present value of harvest revenue over infinite rotations

W^J the net present value of harvest revenue plus the present value of amenity services for a single rotation

W the net present value of harvest revenue plus the present value of amenity services over infinite rotations

E the present value of amenity services over infinite rotations

Appendix 1. Proof of Lemma 1

Notice first that $pf(T) - rU$ can be re-expressed as

A.1 $$pf(T) - rU = \int_0^T pf(s)e^{-rs}ds \left[\frac{pf(T)}{\int_0^T pf(s)e^{-rs}ds} - \frac{r}{(1-e^{-rT})} \right]$$

Given that $f'(T) > 0$ at the economically relevant rotation cycles, one has

$$pf(T)\int_0^T e^{-rs}ds > \int_0^T pf(s)e^{-rs}ds \Leftrightarrow pf(T)(\frac{1-e^{-rT}}{r}) > \int_0^T pf(s)e^{-rs}ds$$

$$\Leftrightarrow \frac{pf(T)}{\int_0^T pf(s)e^{-rs}ds} > \frac{r}{1-e^{-rT}}.$$ **Q.E.D.**

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Appendix 2. Proof of Lemma 2

Recall first that the expression $F(T) - rE = F(T) - r(1-e^{-rT})^{-1}\int_0^T F(s)e^{-rs}ds$

can be rewritten as

A.2 $$\int_0^T F(s)e^{-rs}ds \left[\frac{F(T)}{\int_0^T F(s)e^{-rs}ds} > (\leq) \frac{r}{(1-e^{-rT})} \right].$$

If $F'(T) > (\leq)\ 0$, then $\int_0^T F(T)e^{-rs}ds > (\leq) \int F(s)e^{-rs}ds$

$$\Leftrightarrow \frac{F(T)}{r}(1-e^{-rT}) > (\leq) \int_0^T F(s)e^{-rs}ds$$

$$\Leftrightarrow \frac{F(T)}{\int_0^T F(s)e^{-rs}ds} > (\leq) \frac{r}{(1-e^{-rT})}.$$ Hence, we have

$$F(T) - rE = \int_0^T F(s)e^{-rs}ds \left[\frac{F(T)}{\int_0^T F(s)e^{-rs}ds} > (\leq) \frac{r}{(1-e^{-rT})} \right] > (\leq)\ 0,$$ so that

$F(T) - rE > (\leq)\ 0$ as $F'(T) > (\leq)\ 0$. **Q.E.D.**

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Appendix 3. Proof of Proposition 2.

The first-order condition (12) of the text can be rewritten as follows:

A.3 $\quad W_T(\tau, t) = 0 \iff$

$$\hat{p}(f'(T) - rf(T) - rf(T)(e^{rT} - 1)^{-1}) + rc(1 - e^{-rT})^{-1} + F(T) - rE = 0$$

Therefore

A.4 $\quad \left[f'(T) - rf(T) - rf(T)(e^{rT} - 1)^{-1}\right] > (\leq)\ 0$ as $rc(1 - e^{-rT})^{-1} + F(T) - rE < (\geq)\ 0$

Differentiating the first-order condition A.3 with respect to τ and t gives

A.5 $\quad W_{T\tau}(\tau, t) = -p\left[f'(T) - rf(T) - r(e^{-rT} - 1)f(T)\right]$, and $W_{Tt}(\tau, t) = \dfrac{W_{T\tau}(\tau, t)}{p}$.

Comparing equation A.4 and A.5 gives the Proposition 2. **Q.E.D.**

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